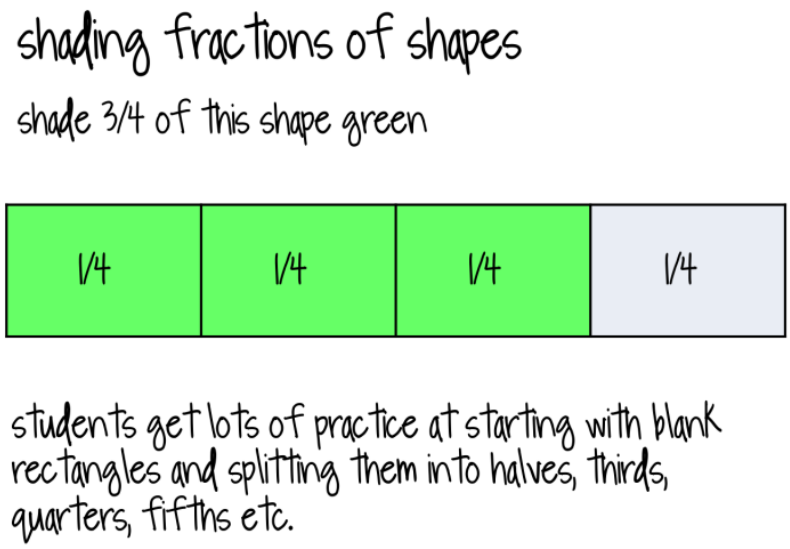
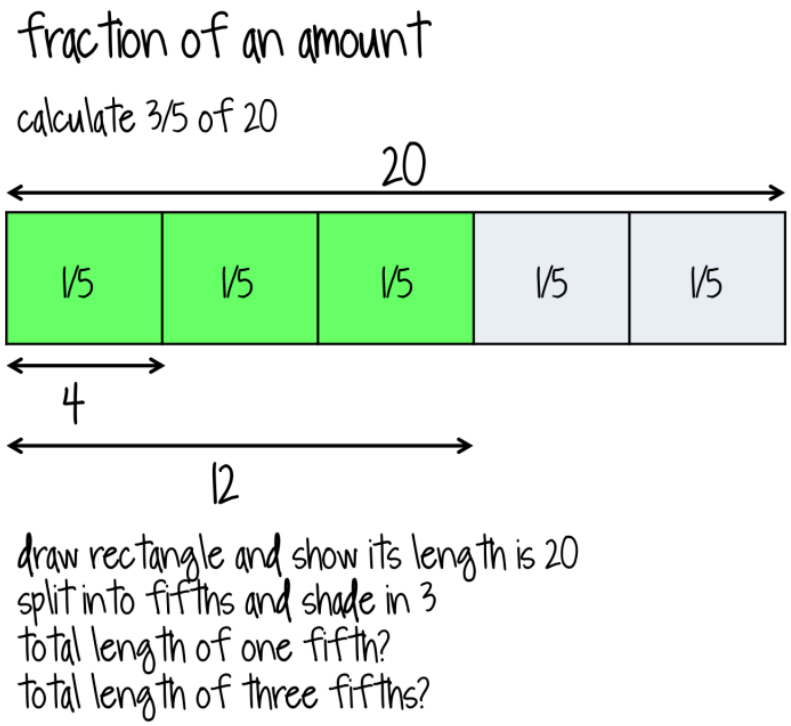
To start with students are given blank (bar) rectangles (on plain paper) and then get used to dividing the bars into halves, thirds, quarters etc:



They can then calculate a fraction of a quantity by first drawing the fraction in the bar, showing the length of the bar to be the quantity and then calculating the length of the shaded part:



Again, you'll end up at 'divide by denominator, multiply by numerator' eventually, but this does show the concept of what's going on very nicely and is a good route into showing where the algorithm comes from.

Next up, equivalent fractions:

equivalent fractions

find equivalent fractions to 2/5

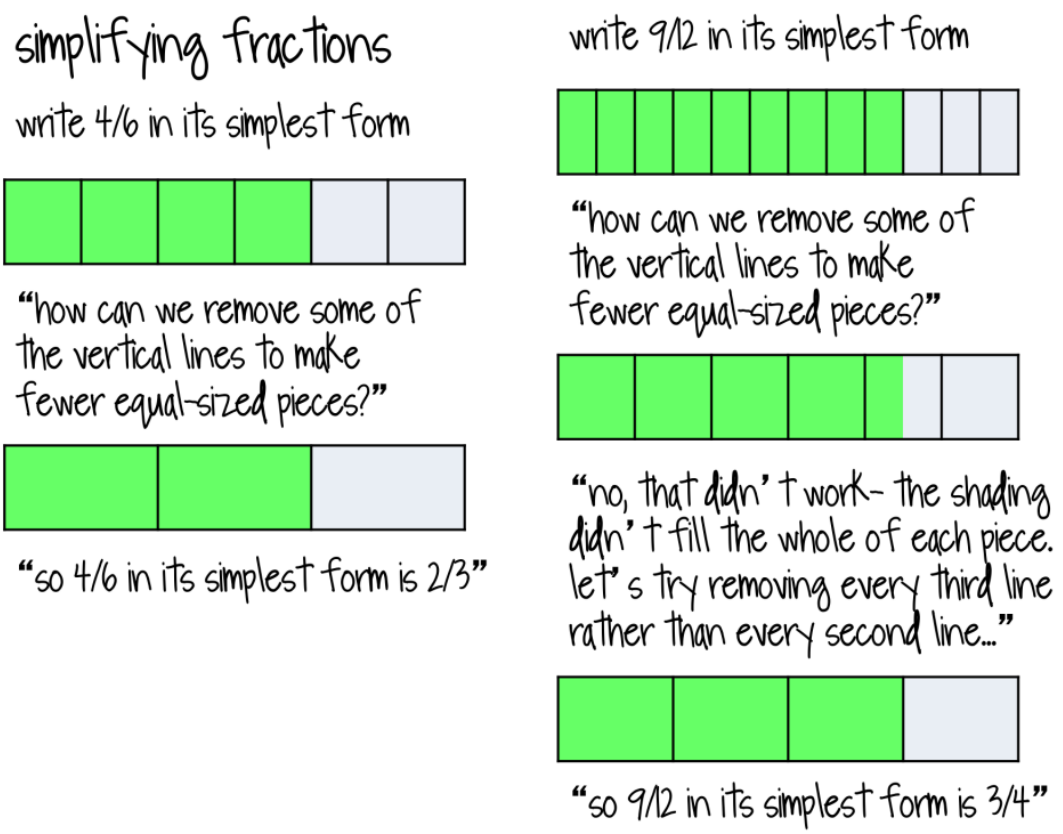
"lets take each fifth and split it into two pieces (students draw in these division lines)"

"so 4/10 is equal to 2/5"

"lets take each fifth and split it into three pieces (students draw in these division lines)"

"so 6/15 is equal to 2/5 as well"

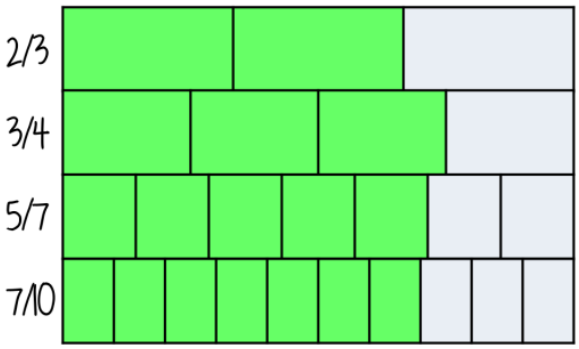
Then simplifying fractions:



A 'fraction wall' (as many teachers use traditionally in England) can be used for ordering fractions:

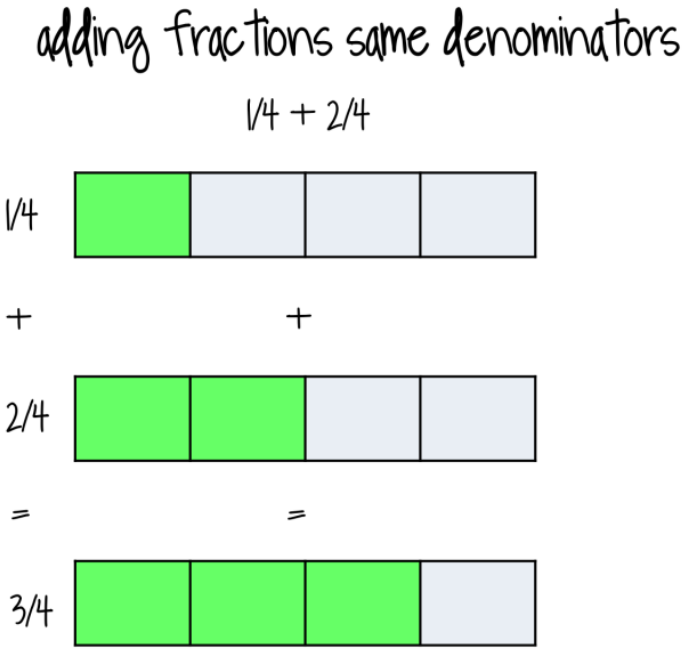
ordering fractions

order these fractions by size from the smallest to the largest: 2/3, 3/4, 5/7, 7/10



2/3, 7/10, 5/7, 3/4

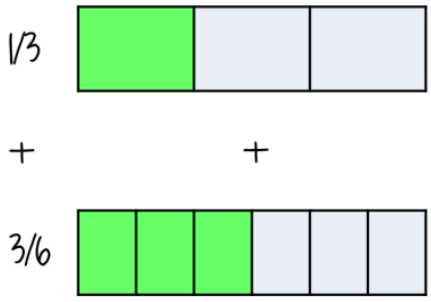
Adding fractions with the same denominators:



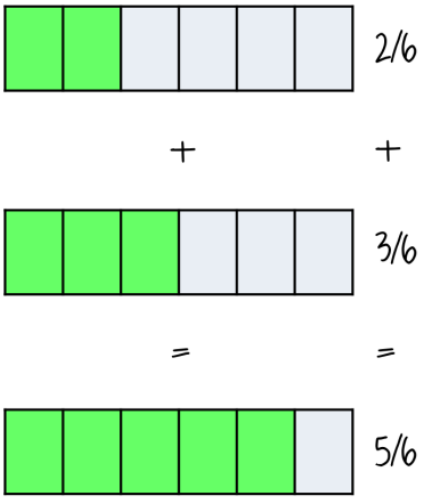
Adding fractions with different denominators:

adding fractions different denominators

1/3 + 3/6

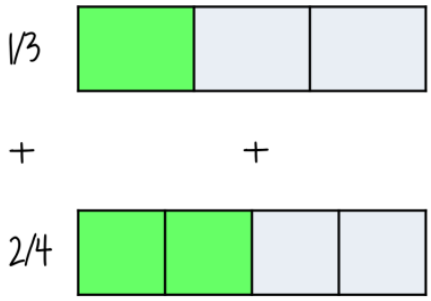


"let's turn this one into sixths." we split the thirds into sixths and the diagrams become as per those on the right

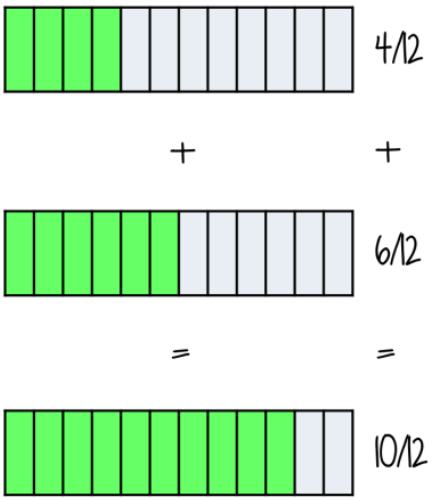


adding fractions different denominators

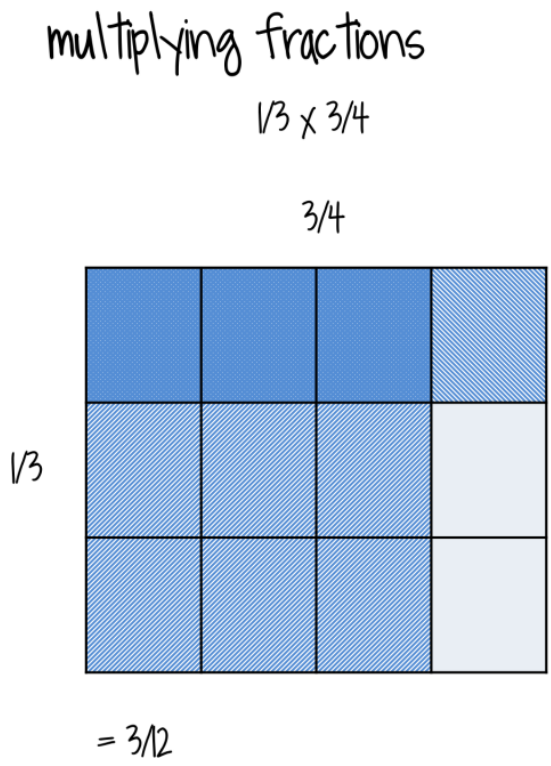
1/3 + 2/4



"let's turn both into twelfths." we split both bars into twelfths and the diagrams become as per those on the right



Multiplying fractions:



Dividing by fractions (works ok so long as you have integer answers). That’s enough to get across what is going on… Then you can lead into the method…

dividing integers by fractions

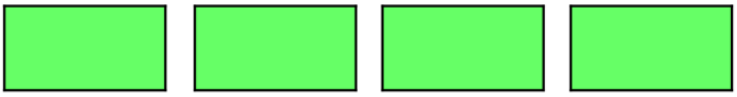
$3 \div 1/2$

"how many 1/2 s go into 3 whole ones?"

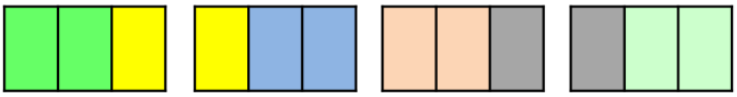
$= 6$

dividing integers by fractions

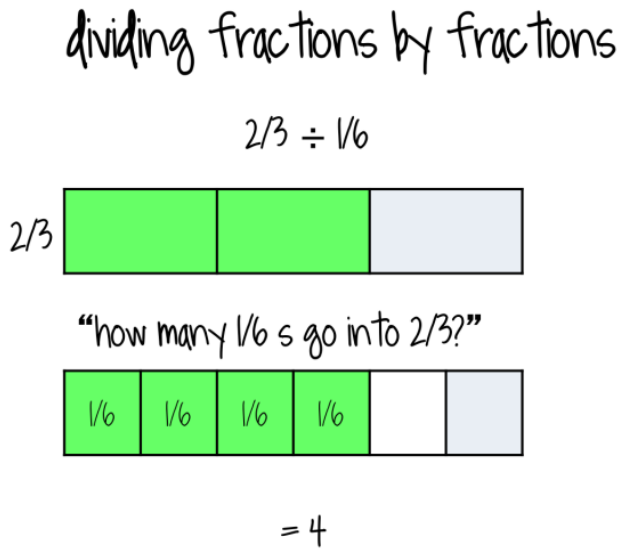
$4 \div 2/3$



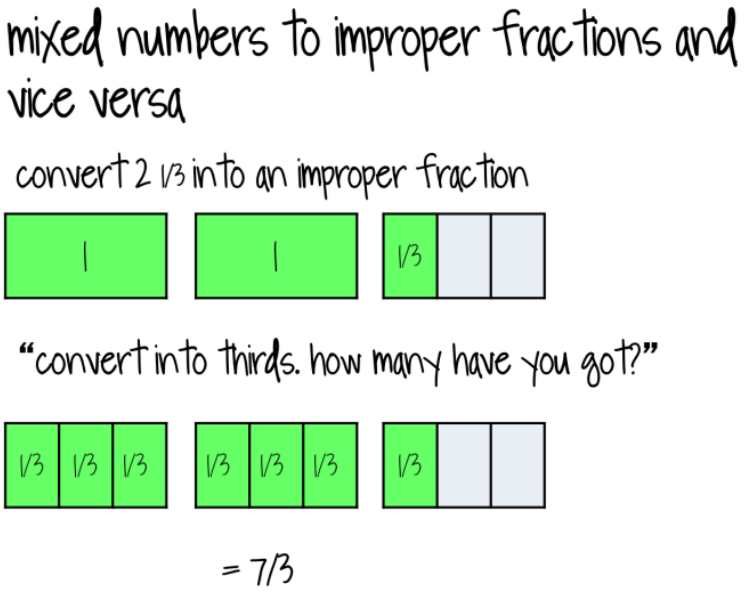
"how many 2/3 s go into 4 whole ones?"



$= 6$



Converting mixed numbers and improper fractions:



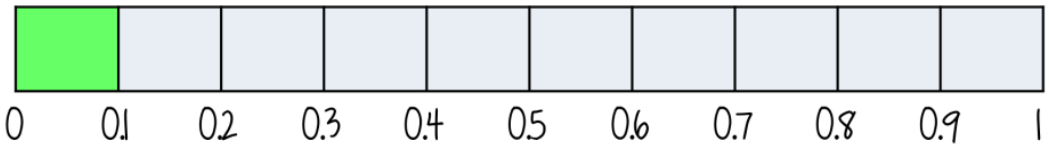
Next up, understanding place value in decimal numbers. This approach lets you deal with lots of misconceptions like 0.62 not being larger than 0.7 etc. ***Importantly, this is now taking the bar model and putting a decimal number line onto it.*** This forms the basis for many topic models that follow:

place value in decimal numbers

units	tenths	hundredths
0	. 6	2

"this is what a whole unit looks like"

"this is what a tenth looks like"

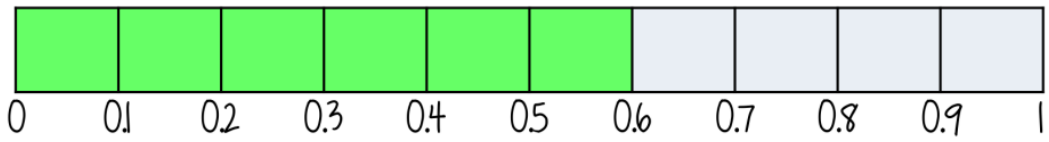


"this is what a hundredth looks like. gosh, aren't they small?!"

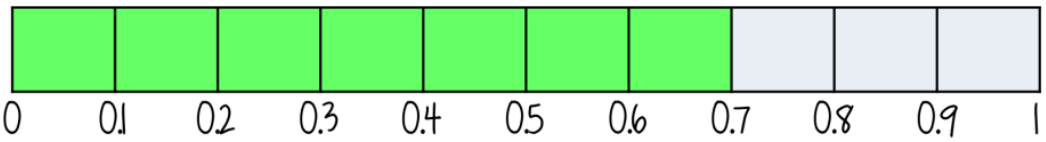
place value in decimal numbers

units	tenths	hundredths
0	. 6	2

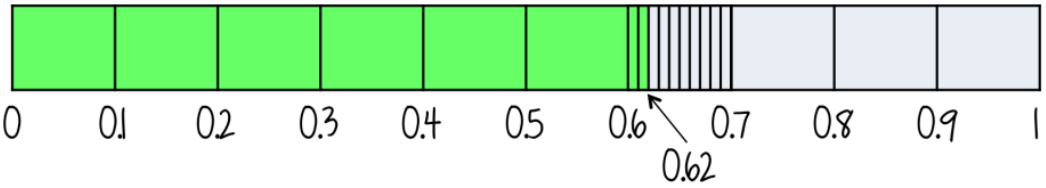
"so 0.6 looks like this..."



"and 0.7 would look like this...



"0.62 would look like this. it's larger than 0.6 but smaller than 0.7"

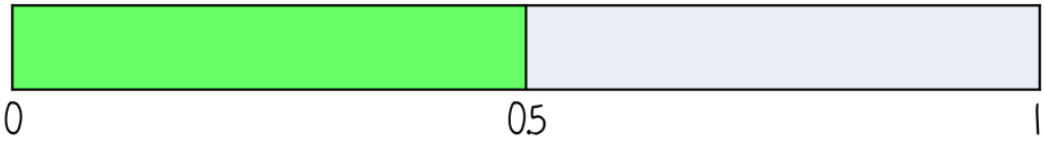


Now they have an understanding of how the decimal number line works, and they can draw bar models for fractions, they can combine the two on one diagram to convert between fractions and decimals:

fractions to decimals and vice versa

now students have an understanding of the decimal number line, they can put it onto a bar model to convert between fractions and decimals. draw the bar model fraction first then put on the decimal number line identifying the increments by dividing 1 by the denominator of the fraction

1/2 = 0.5



3/10 = 0.3

2/5 = 0.4

fractions to decimals and vice versa

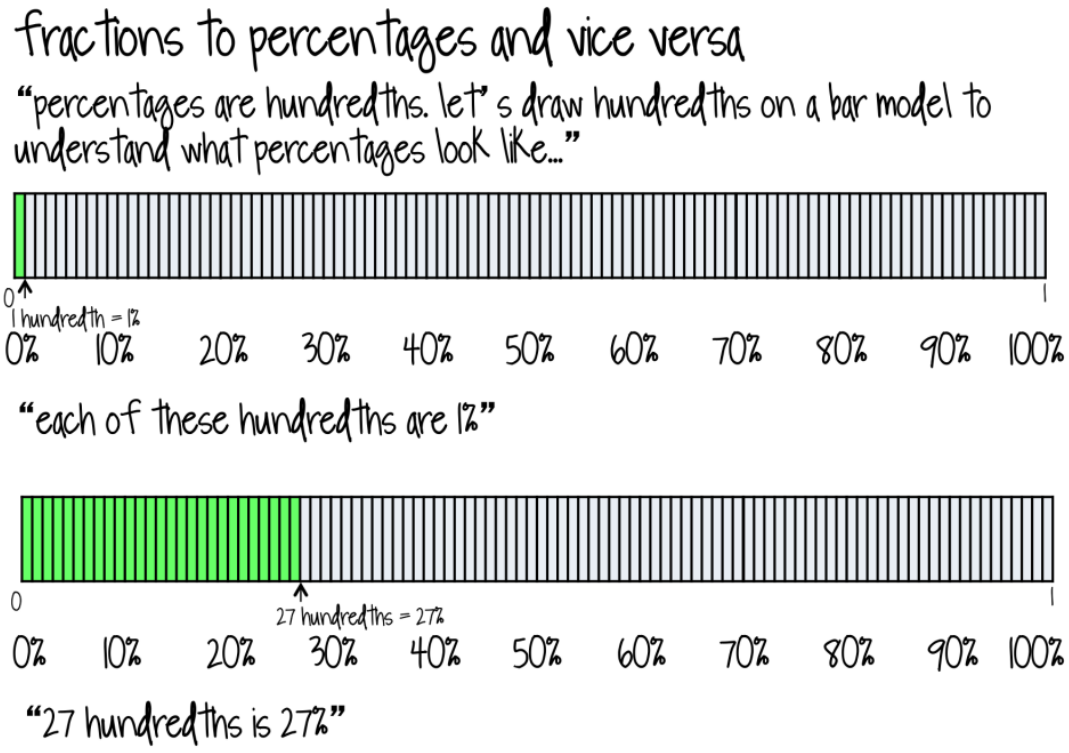
another way of thinking of fractions showing where "to convert from fraction to decimal divide numerator by denominator" comes from

convert 3/5 to a decimal

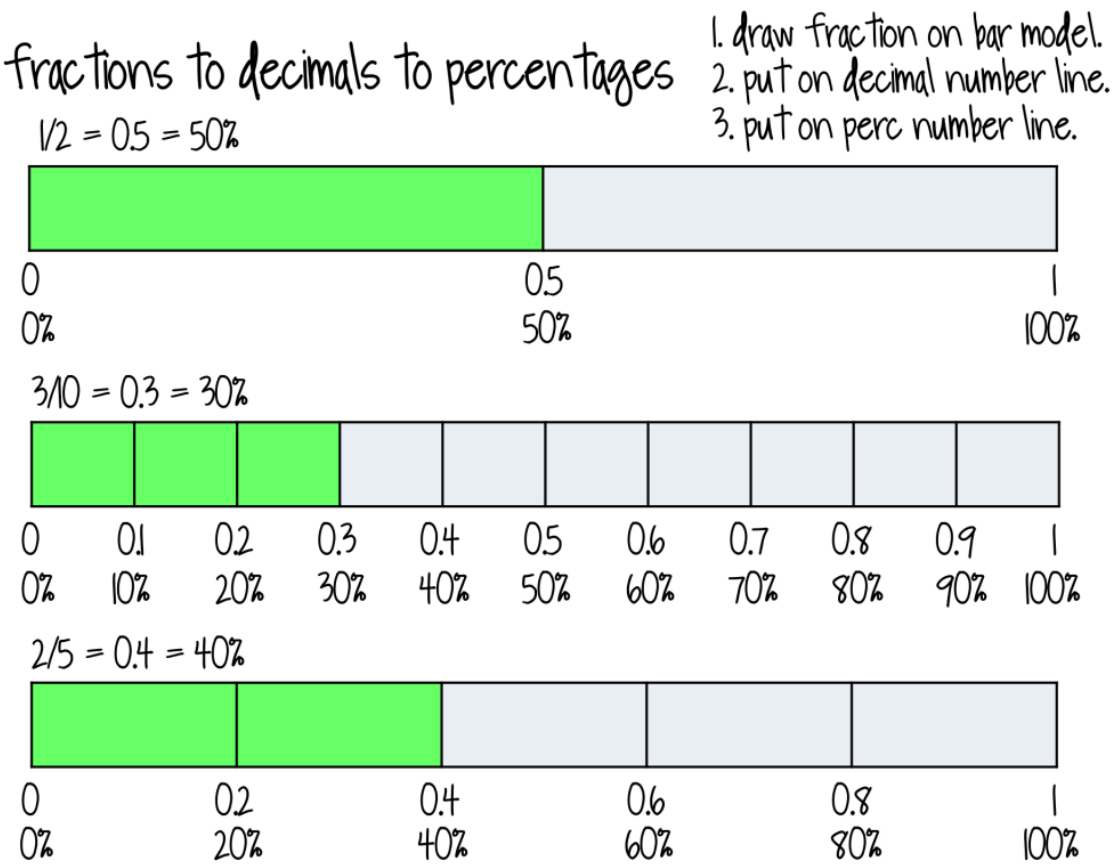
split 3 whole ones up into fifths (divide by 5)

"gives us the same answer as when we started with a whole unit, divided by 5 then multiplied by 3"

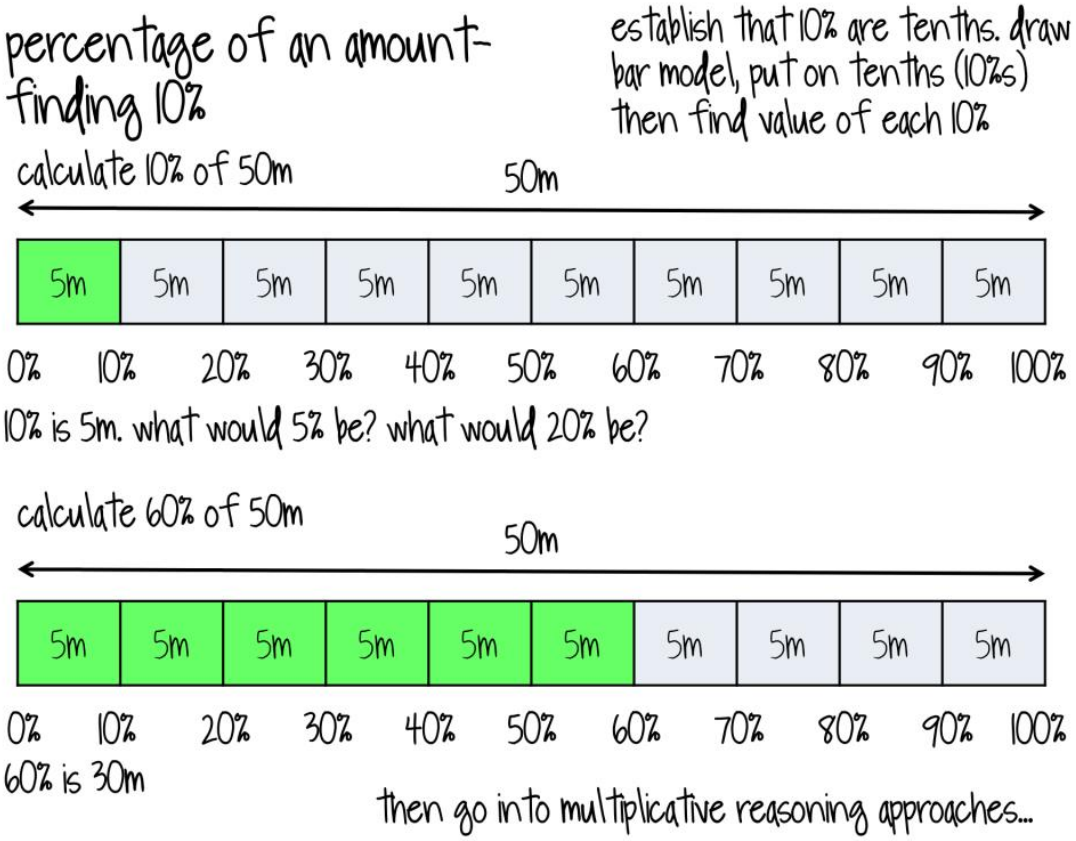
Next they can learn that percentages are hundredths, and in doing so can put a***percentage number line*** under the bar model:



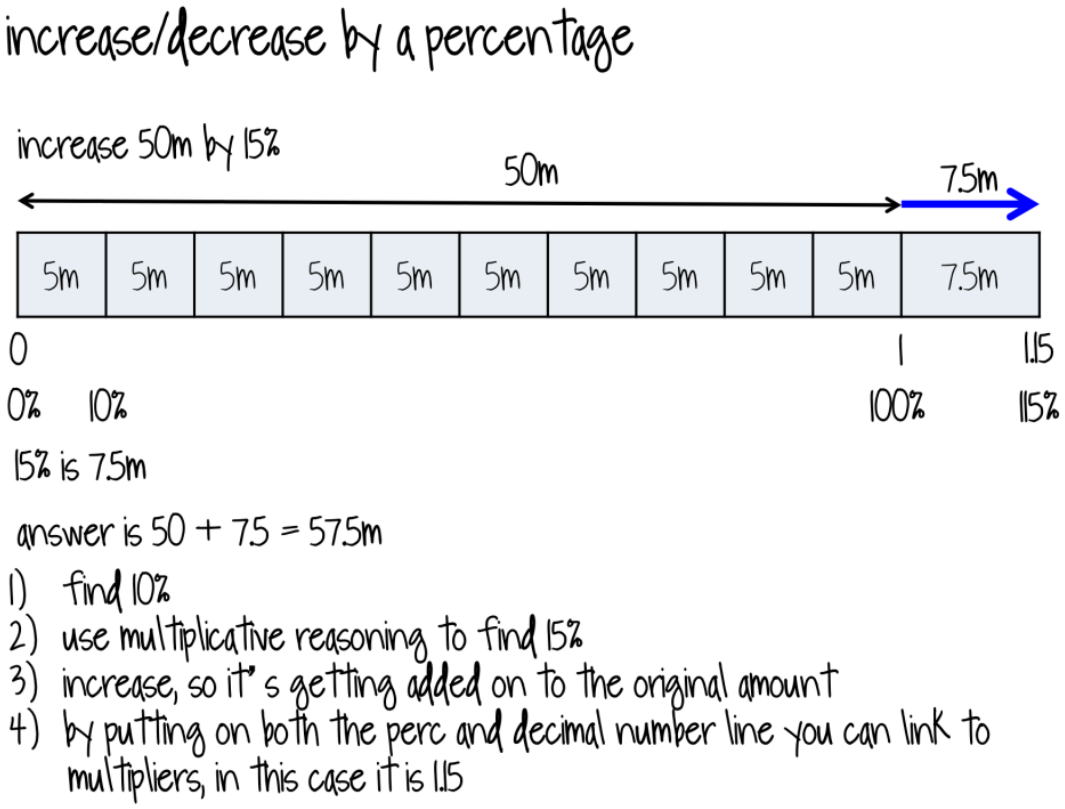
They can now combine the fractional bar model with both the decimal and percentage number lines directly underneath it to convert between fractions, decimals and percentages. They draw the fraction bar first, then put on the decimal increments (by dividing 1 by the denominator) and finally put on the percentages (by dividing 100 by the denominator):



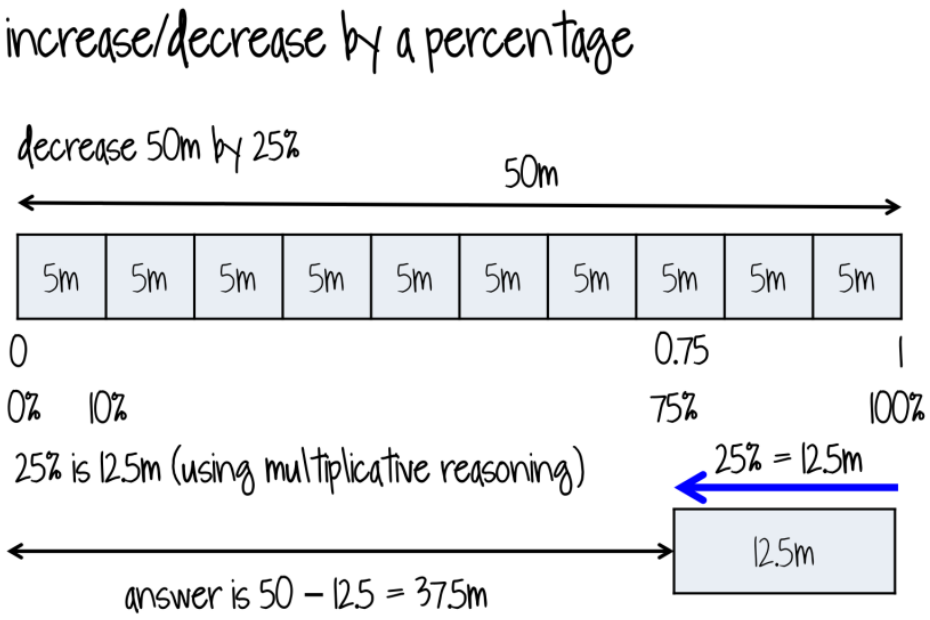
Similarly to fraction of an amount earlier, you can use this approach to introduce percentage of an amount. Starting with showing how to find 10% of a number (and thus why you divide by 10), it serves as a nice way into multiplicative reasoning approaches to ‘build off the 10%’ to find other percentages:



Other percentage topics then follow, such as percentage increase and decrease:

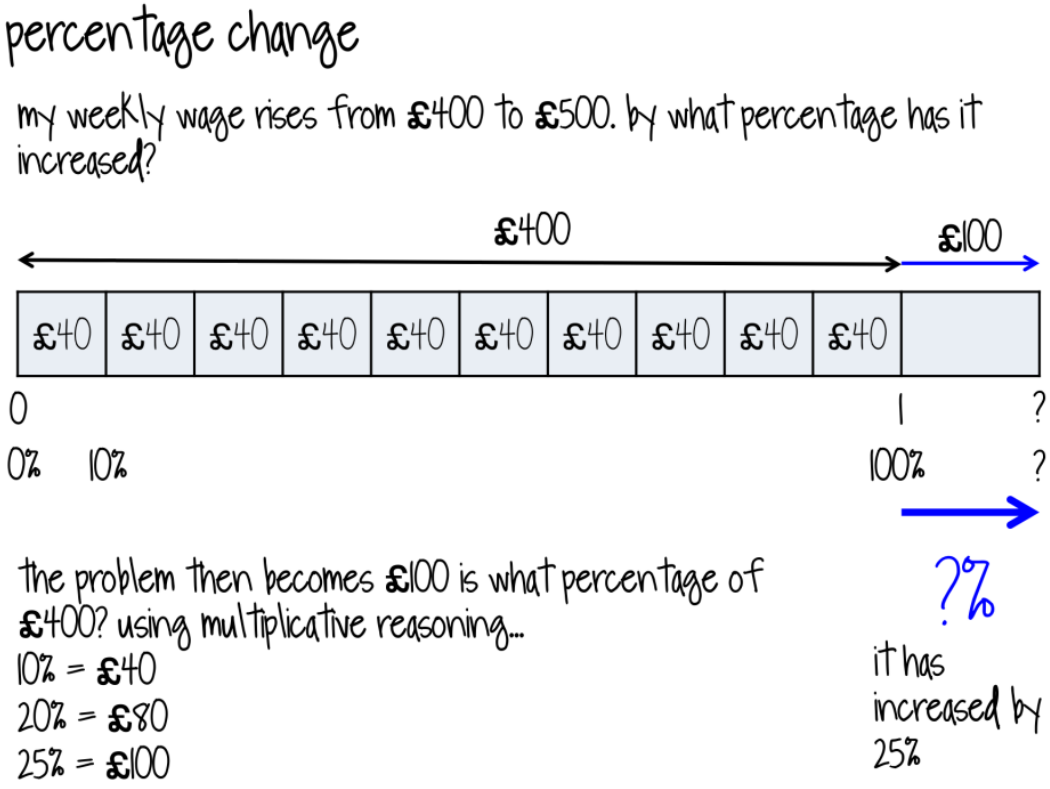


By putting both the decimal and percentage number lines on the bar model for this, you can clearly show where percentage multipliers come from, including the ones less than unity for percentage decreases:

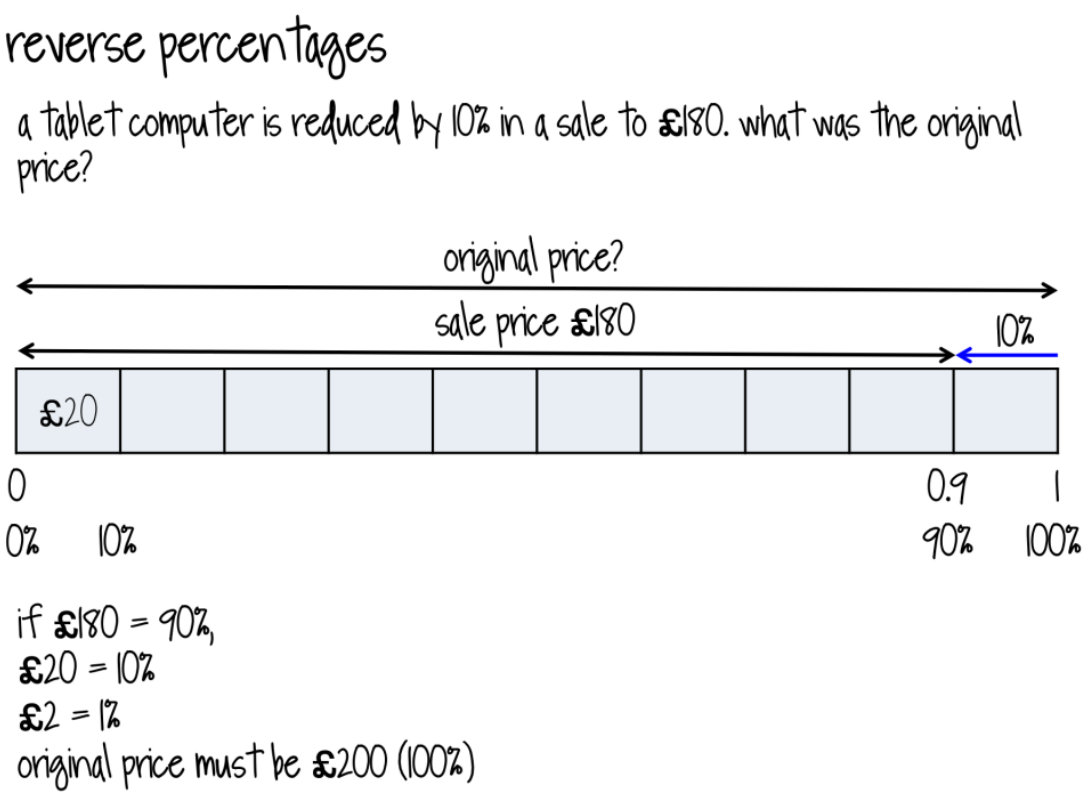


by putting on both the perc and decimal number line you can link to multipliers, in this case it shows why the multiplier for a 25% decrease is 0.75

You can introduce calculating a percentage change using the bar model approach:



It's a particularly nice diagrammatic way in to teaching reverse percentages:



Once the above techniques have been mastered, it can be used for showing how compound interest works as follows. Particularly nice is if you turn the bars so they are vertical, it shows the exponentially increasing relationship:

It works for ratio too:

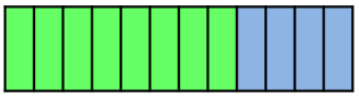
equivalent and simplifying ratios

1 : 2

is equivalent to...

2 : 4

8 : 4

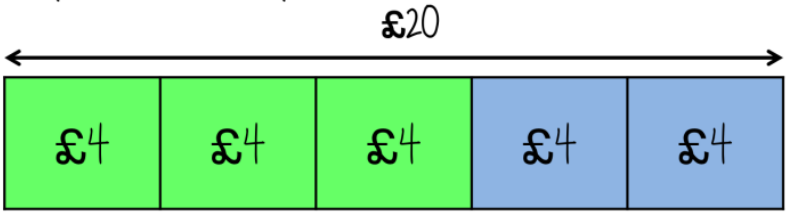


simplifies to...

2 : 1

sharing a quantity in a given ratio

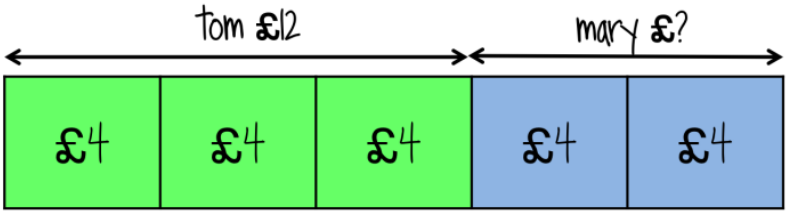
share £20 in the ratio 3 : 2



draw bar model showing ratio 3: 2 and total length £20
find 1 part is £4
answer is £12 : £8

sharing a quantity in a given ratio when you are told one side of the ratio, not the whole amount

tom and mary share some money in the ratio 3 : 2. tom gets £12. how much does mary get?



draw bar model showing ratio 3: 2 and tom getting £12
find 1 part is £4
mary gets £8